Stage 1 Mathematics Lesson Plan

SACE TOPIC 1.4: Composite Functions

# Overview

### Time Required

55 minutes

### Instructional Strategies

Teacher demonstrations, class discussions, practice questions.

### Materials

* Whiteboard or document camera
* Haese and Harris Year 11 Math Methods Text Book: Chapter 3D (pages 75-76) or equivalent.

### Assumed Knowledge

* Students know function notation
* Students know how to substitute a value into a function
* Students can expand out brackets
* Students can work out the domain of a function

# Objectives:

Students will **understand:**

* That the x in f(x) can be anything – including another function
* Functions don’t have to start with ‘y’ or ‘f’

Students will **know:**

* The words: compose and composition in a mathematical sense
* The notation f ○ g (f composed with g)
* The notation f(g(x))
* The domain of f ○ g is the set of all real numbers x in the domain of g such that g(x) is in the domain of f

Students will **be able to:**

* Compose functions algebraically

# Procedure

### Introduction: Check the assumed knowledge (5 minutes)

Quick demonstration on substituting known values into a function of one variable and then calculate the domain. Come up with 5 basic examples for students to work through to get the class settled. Make sure examples use functions of different names (f(x), s(t), y(x), etc.)

 Understand: Functions don’t have to start with ‘y’ or ‘f’.

### Motivating Example: A physical machine analogy (10 minutes)

Suppose there are two machines. A plasma cutter (cuts metal sheets) and a drill press (drills holes in wood and metal). We might say that each machine ‘is’ or performs one function

*Ask:**what the domain (types of acceptable material) for each machine would be.*

Now suppose we were to make a super machine that combines cutting and drilling together. We say we compose the machines. Suppose the drilling happens first, then the cutting, i.e. the output of the drilling machine is the input of the cutting machine.

*Ask: What is the domain of this composite machine.* The answer is metal because once the drilling is complete, the plasma cutter will light the wood on fire.

*Ask: Would the domain change if the cutting came first?* The answer is no because the cutter only takes metal in the first place.

Understand: The domain of a composition is the intersection of both function domains.

### Introduce Composite Function Notation (5 minutes)

On page 75, students will see this formal definition. Direct students to it or write it on the board.



*Explain in English*: “Given some function f that takes a value x and maps it to some UNIQUE value f(x) and some function g which takes a value x and maps it to some UNIQUE value g(x), the composite function of f composed with g will take the output of g(x) and use it as the input to f(x). That is, f(g(x)).”

The emphasis on ‘unique’ is there to highlight that this is a function, not a relation.

*Highlight on the board*: f ○ g = f(g(x)) and g ○ f = g(f(x)).

 Know: The words: compose and composition in a mathematical sense

 Know: The notation f ○ g (f composed with g)

 Know: The notation f(g(x))

### Composition Demonstration (5 minutes)

Come up with a simple f(x) and g(x) and have the class evaluate g(0), g(1), g(5) and then evaluate f(g(0)), f(g(1)), f(g(5)) but with their previous answers, not the function. Next, demonstrate how to compose f and g such that we get f(g(x)). Have the students calculate f(g(0)), f(g(1)), f(g(5)) with the composed function and have them compare it with their previous calculations.

 Able To: Compose functions algebraically

 Understand: That the x in f(x) can be anything – including another function

### Solo Practice (20 minutes)

Have students complete exercise 3D on page 76 of the text book or find equivalent composition questions. The teacher should walk around and assist students and correct misunderstandings.

### Domains of Composite functions: A Mathematical Parallel to The Machines (10 minutes)

Have the class recall the physical machine analogy and the domains. A math parallel to the drilling machine might be f(x) = x2 and the cutting machine, g(x) = sqrt(x).

The domain of f is all real numbers (metal and wood), the domain of g is all non-negative real number (metal).

*Ask the class to compose the function f(g(x))* – that is, the drilling machine has a cutting machine inside that first cuts and then gives the cut material to the driller. The result is f(g(x)) = (sqrt(x))2 = x

*Ask the class to find the domain of f(g(x))*. They’ll probably say all real numbers because some function h(x) = x has the domain of all real numbers. Recall the start of the class where substituted values into the inner function and then put the result in the outer function. If they try the same approach here with g(-1), they’ll find a problem at sqrt(-1). This doesn’t seem like it would be a problem when we see f(g(x)) = x

*Know:* The domain of f ○ g is the set of all real numbers x in the domain of g such that g(x) is in the domain of f.

When we look at the driller with an internal cutter machine, we see the driller as being able to take wood and metal but we have to respect the domain of the internal cutter machine which is metal only. This is like f(g(x)).

Note to teachers: The analogy doesn’t work for g(f(x)). This is because the cutter maps metal to metal and the driller maps metal to metal OR wood to wood. The function f(x) = x2 maps real numbers to real non negative numbers which is like the driller mapping wood and metal to metal. Know this in case a student starts to get inquisitive about it. Students will cover it in much more depth in first year university math.

# Assessment

A few questions on composition will be in the topic test.

# Homework

Nothing set – make sure all questions in the exercise are completed before the next class.